The other natural transformation from basic linear algebra which I'd like to say something about is actually between two functors

$$\mathbf{Field} \to \mathbf{Group}$$

Which might seem surprising, until I tell you what they are.

First of all, if F is any field, we can form the group of $n \times n$ invertible matrices with entries in F, commonly known as the general linear group, or $\mathrm{GL}_n F$. Moreover, if $\phi : F \to K$ is any field homomorphism, then we get an induced group homomorphism

$$\mathrm{GL}_n(\phi) : \mathrm{GL}_n F \to \mathrm{GL}_n K$$

which simply applies the field homomorphism to each of the entries of the matrix. It's not hard to check that this is actually a functor

$$\mathrm{GL}_n : \mathbf{Field} \to \mathbf{Group}$$

from the category of fields to the category of groups.

Secondly, if F is any field, we can take its group of units (invertible elements) under multiplication F^*, which, since it's a field, is everything except for 0. So long as you're familiar with the definitions of these things, it's easy to see that any field homomorphism induces a group homomorphism directly, and that the functor laws are obeyed. So we have:

$$* : \mathbf{Field} \to \mathbf{Group}$$

In addition to these, we have for any field F, a group homomorphism

$$\det_F : \mathrm{GL}_n F \to F^*$$

the determinant! It's a group homomorphism now because of the basic results from linear algebra that:

$$\begin{aligned} \det(\mathbf{1}) &= 1 \\ \det(AB) &= \det(A)\det(B) \\ \det(A^{-1}) &= (\det(A))^{-1} \end{aligned}$$

Now, the naturality square,

$$\begin{array}{ccc} \mathrm{GL}_n F & \xrightarrow{\mathrm{GL}_n \phi} & \mathrm{GL}_n K \\ {\scriptstyle \det_F}\downarrow & & \downarrow{\scriptstyle \det_K} \\ F^* & \xrightarrow{\phi^*} & K^* \end{array}$$

commutes because the determinant is a polynomial in the entries of the matrix, and field homomorphisms commute with any polynomial map.

So the determinant too numbers among natural transformations.